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PERTURBATION FINITE ELEMENT METHOD FOR REFINING MAGNETIC CIRCUIT MODELS

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***Abstract*— Model refinements of magnetic circuits are performed via a subproblem finite element method based on a perturbation technique. An approximate problem considering ideal flux tubes is first solved. It gives the sources for finite element perturbation problems considering all the details of the exterior regions, thus accounting for leakage fluxes. The procedure simplifies both meshing and solving processes. It quantifies the gain given by each model refinement on both local and global quantities.**

I. INTRODUCTION

The perturbation of finite element (FE) solutions provides clear advantages in repetitive analyses [1], [2] and helps improving the solution accuracy [3]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and increases computational efficiency thanks to the reduced size of each subproblem.

A perturbation FE method is herein developed for refining the magnetic flux distribution in magnetic circuits starting from simplified FE models based on ideal flux tubes [4]. These are then perturbed to account for leakage fluxes of different natures (near air gaps, near inductors, in slots, etc.). The developments are performed for the magnetic vector potential FE magnetostatic formulation, paying special attention to the proper discretization of the constraints involved in each subproblems. The method is applied and validated on test problems.

II. A SERIES OF COUPLED SUBPROBLEMS

A. Canonical problem in a strong form

A canonical magnetostatic problem p is defined in a domain Ω_p, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. Subscript p refers to the associated problem p. The equations, material relations, boundary conditions (BCs) and interface conditions (ICs) of problem p are

$$\operatorname{curl} \boldsymbol{h}_p = \boldsymbol{j}_p\,, \quad \operatorname{div} \boldsymbol{b}_p = 0\,, \quad \boldsymbol{b}_p = \mu_p\, \boldsymbol{h}_p + \boldsymbol{b}_{s,p}\,, \qquad \text{(1a-b-c)}$$

$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = 0\,, \quad \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = 0\,, \qquad \text{(1d-e)}$$

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = \boldsymbol{j}_{su,p}, \quad [\boldsymbol{n} \cdot \boldsymbol{b}_p]_{\gamma_p} = \boldsymbol{b}_{su,p}, \qquad \text{(1f-g)}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{j}_p$ is the prescribed current density, μ_p is the magnetic permeability and $\boldsymbol{n}$ is the unit normal exterior to Ω_p.

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The field $\boldsymbol{b}_{s,p}$ is a possible volume source, usually used for fixing a remnant induction. The notation $[\cdot]_\gamma = \cdot|_{\gamma^+} - \cdot|_{\gamma^-}$ expresses the discontinuity of a quantity through any interface γ (with sides γ^+ and γ^-) in Ω_p, which is allowed to be non-zero. The associated surface fields $\boldsymbol{j}_{su,p}$ and $\boldsymbol{b}_{su,p}$ are generally zero, defining classical essential or natural ICs for the physical fields. If nonzero, they define possible surface sources. A key element of the developed method is to determine the volume and surface sources of problem p from parts of solutions of other problems.

B. Each subproblem defines a perturbation

The objective is solving successive problems, the addition of their solutions $\mathbf{u}_p$ giving the solution $\boldsymbol{u}$ of a complete problem (with $\boldsymbol{u} \equiv \boldsymbol{h}, \boldsymbol{b}, \ldots$). At the discrete level, each problem is defined in its own domain and mesh, which decreases the problem complexity and allows distinct mesh refinements. Also, such a superposition of solutions allows each subproblem to satisfy constraints and relations that are not necessarily shared with the complete problem. Consequently, each subproblem is generally perturbed by all the others and each solution has to be calculated as a series of corrections. The calculation of the correction $\mathbf{u}_{p,i}$ in a problem p,i is kept on till convergence up to a desired accuracy. It must account for the influence of the previous corrections $\mathbf{u}_{q,j}$ of the other subproblems (via their projection from mesh q to mesh p), with j the last iteration index for which a correction is known. Initial solutions $\mathbf{u}_{p,0}$ are set to zero. The iterative process is required when a correction becomes a significant source for any of its source problems, which is inherent to large perturbation problems. In addition to the iterations between subproblems, classical inter-problem iterations are needed in nonlinear analyses. The global quantities linearly related to each correction (fluxes and magnetomotive forces [4]) are to be added to obtain their complete values.

A change of the permeability in a volume region (from μ_q for problem q to μ_p for problem p), due to either the change of properties of existing materials or the addition or suppression of materials, has already been shown to generate a volume source (or a region-type source) in the associated material relation [2], i.e. $\boldsymbol{b}_{s,p} = (\mu_p - \mu_q)\, \boldsymbol{h}_q$. Taken to certain limits, such changes have to be expressed via surface sources, as shown hereafter.

C. Perturbations: from ideal to real flux tubes

In a first problem $p = 1$, the magnetic flux is forced to flow only in a subregion with perfect flux walls, i.e. a set of flux tubes $\Omega_1 = \Omega_{ft,1}$ of the whole domain Ω (complete problem).

Other problems $p > 1$ consider then some flux walls become permeable (portion by portion). This allows leakage flux in some exterior regions $\Omega \backslash \Omega_1$ and leads to a change of the flux distribution in Ω_1. Solution refinements are thus achieved. The differential equations (1a) and (1b) remain unchanged for each subproblem, but the problems differ by their sources, in particular via (1f).

In problem 1, the ideal flux tubes are considered with a zero normal magnetic flux density BC on their boundaries $\Gamma_{ft,1} = \partial\Omega_1$. The trace of the magnetic field is unknown on $\Gamma_{ft,1}$. Once determined from the solution in Ω_1, it can be used as a BC for calculating the solution in $\Omega \backslash \Omega_1$, with all the precise characteristics of this exterior region (e.g., inductors and other regions). This task is however avoided, preferring the magnetic field to be simply zero in $\Omega \backslash \Omega_1$. For that, problem 1 gathers all the inductor parts of the exterior region inside the double layer defined by $\Gamma_{ft,1}{}^{+}$ and $\Gamma_{ft,1}{}^{-}$, the inner and outer sides of $\Gamma_{ft,1}$ with regard to Ω_1; this defines idealized inductors. Each problem $p > 1$ must then correct the already obtained solutions, in particular solution 1, via particular ICs corrections. Such ICs are surface sources (or interface-type sources) fixing the possible trace discontinuities of $\boldsymbol{h}_p$ and $\boldsymbol{b}_p$ in terms of the solution of other problems q.

All the constraints involved in the subproblems have to be carefully defined in the associated FE formulations, respecting their inherent strong and weak natures. Adapted magnetic vector potential formulations will be developed in the extended paper. As a result, an efficient and accurate computation of local fields and global quantities (e.g., flux, MMF, reluctance) is obtained.

The sources of each problem $p > 1$ are initially supported by other meshes q. They are transferred via a projection method to mesh p, but only in a reduced support limited to the layer of FEs touching $\Gamma_{ft,q}$ in $\Omega_{ft,q}$, as needed by the FE formulation. This naturally reduces the computational efforts of the projection process.

III. Application Example

An electromagnet is considered to illustrate and test the method. It consists of a U-shape core surrounded by a stranded inductor and separated from an I-shape core via two air gaps (Fig. 1). An approximate solution $p = 1$ is first calculated in an idealized flux tube (Fig. 2, *left*), with a fixed magnetomotive force as excitation and a coarse mesh of the tube (Fig. 1, *middle*). This solution serves then as a source for a perturbation problem $p = 2$ allowing leakage flux in the inner region of the core (Fig. 2, *middle left and right*), considering either an idealized inductor (limited to the upper branch of the U-core) or the actual one. Another problem $p = 3$ then allows leakage flux in the outer region (Fig. 2, *right*). These problems calculate the actual flux distribution in the related inductor portions and in the vicinity of the gaps, with their own adapted meshes, naturally offering a zoom on the leakage fluxes and their sources (otherwise usually hidden in the complete solution). They also correct the inductor flux linkage (Fig. 3), and consequently the reluctances. The way to select and define some reduced domains for the subproblems will be also discussed in the extended paper.

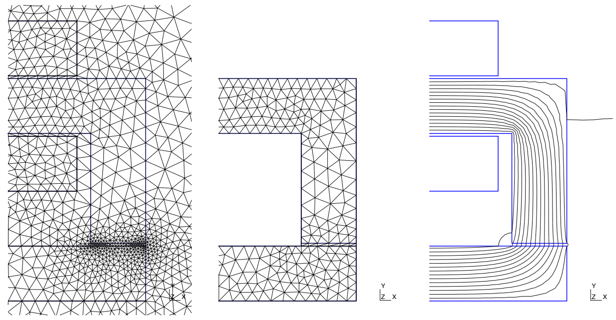

Fig. 1. Meshes (half portions) of the whole studied domain (*left*) and the ideal flux tube (*middle*); field lines of the complete solution (***b***, *right*).

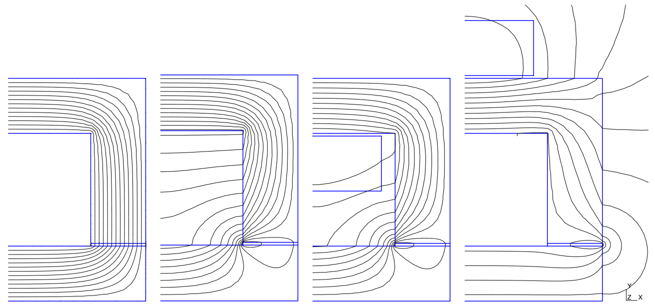

Fig. 2. Field lines in the ideal flux tube ($\boldsymbol{b}_1$, *left*) and in the perturbation problems: with the inner ($\boldsymbol{b}_2$, *middle*; with idealized (*middle left*) or actual coil (*middle right*)) and outer ($\boldsymbol{b}_3$, *right*) leakage fluxes; zoom on the perturbations: the perturbation flux flowing between two consecutive field lines is 7 times lower than the source flux in the ideal tube.

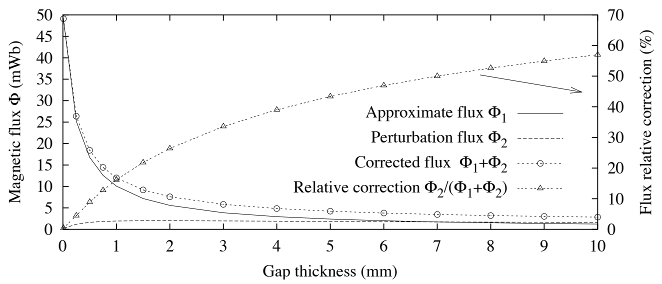

Fig. 3. Importance of the perturbation flux versus the air-gap thickness for the inductor flux linkage (the core width is 20 mm).

IV. Conclusions

The developed perturbation FE method splits magnetic circuit analyses into problems of lower complexity with regard to meshing operations and computational aspects. This allows a natural progression from simple to more elaborate models, while quantifying the gain given by each model refinement and justifying its utility. Additional refinements towards nonlinear behavior, eddy current or 3-D effects are possible extensions.

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